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Massive Excited State of Quantum Gravity as a Dark Particle

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References

• K. Hamada, aXiv:2005.06743

• K. Hamada, ‘’Quantum Gravity and Cosmology Based on Conformal Field Theory’’, (Cambridge Scholars Publishing, 2018)

What Happen Beyond Planck Scale

In Einstein's theory of gravity,
particle with mass exceeding Planck mass is a black hole (BH)

Compton wavelength
= size of particle
$\sim 1/m$

Horizon created by itself
$\sim m/m_{\rm pl}^2$

Particle information is lost

$m < m_{\rm pl}$ $m > m_{\rm pl}$

Describing particle as a point is not justified beyond Planck scale

This is the reason why Planck scale has been recognized
as a wall that cannot be exceeded

➔ Beyond Planck scale, quantization of gravity is necessary
to remove such a classical obstruction

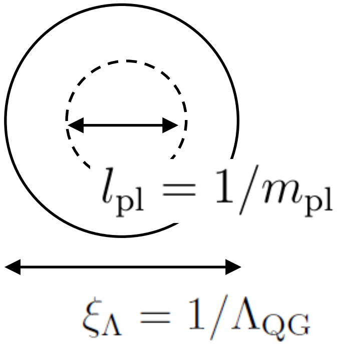
How To Exceed The Wall

Here, I argue that spacetime will transition to a "new phase"
before reaching the wall of Planck scale, and
a quantum world without singularities will come out

As a theory with such property, I proposed
renormalizable quantum theory of gravity formulated
based on a certain conformal field theory,
which suggests the existence of a dynamical scale $\Lambda_{\rm QG}$ that clearly
separates quantum spacetime from Einstein's classical spacetime

In this talk, I will consider an excited state of quantum gravity
with a mass near the Planck mass $m_{\rm pl}$

If $m_{\rm pl} \gg \Lambda_{\rm QG}$, quantum gravity will be activated
before reaching Planck scale, classical horizon
disappears so that such a object can exist



Brief Summary of Renormalizable Quantum Gravity with Background Freedom

Significance of 4th-Derivative Gravity

In general, fourth derivative quantum gravity can resolve the problems existing in Einstein gravity :

- Renormalizablility

  Because coupling constant becomes dimensionless
- Positive-definiteness

  Because curvature squared action is considered,
  while Einstein-Hilbert action is not bounded from below
- No singularities

  When considering an action involving positive-definite Riemann tensor squared, the action diverges for singularities such as Schwarzschild BH ➔ BH is unphysical, unlike Einstein gravity

Nevertheless, simply applying perturbation theory to such a fourth derivative gravity causes the problem of ghosts

To make ghosts unphysical, need to apply a non-perturbative method developed in 2DQG ➔ BRST conformal invariance (=background freedom)

Renormalizable Asymptotically Background-Free Quantum Gravity

$\mathrm{sgn.} = (-1, 1, 1, 1)$

The Action is

$$I = \int d^4x \sqrt{-g} \left[-\frac{1}{t^2} C^2_{\mu\nu\lambda\sigma} - bG_4 + \frac{1}{\hbar} \left(\frac{M^2}{2} R - \lambda + \mathcal{L}_M \right) \right]$$

Weyl tensor Euler density (no *R*^2) conformal inv. in UV

" *t* " is a dimensionless coupling with $\beta_t < 0$ (while *b* is not a dynamical coupling)

Vacuum structure at UV limit ($t \longrightarrow 0$) is given by

$$C_{\mu\nu\lambda\sigma} = 0 \quad \Rightarrow \quad g_{\mu\nu} = \underline{\underline{e^{2\phi}}} \bar{g}_{\mu\nu} = e^{2\phi} \hat{g}_{\mu\lambda} \left(\delta^\lambda{}_\nu + h^\lambda{}_\nu + \frac{1}{2} h^\lambda{}_\sigma h^\sigma{}_\nu + \cdots \right)$$

treated exactly

when doing perturbation cals., $h^\mu{}_\nu \to t h^\mu{}_\nu$

Note that there is no Planck constant in fourth derivative action because gravitational field is exactly dimensionless ➔ purely quantum mechanical

Key Point of Quantization

The action *I* has no fourth-order dynamics of conformal-factor field ϕ

Dynamics (=kinetic term + interactions) of ϕ are induced from the measure:

$$\int [dg \cdots]_{\underline{\underline{g}}}\, e^{iI} = \int [d\phi dh \cdots]_{\underline{\underline{\hat{g}}}}\, e^{i(\underline{\underline{S}}+I)}$$

Nested structure

Practical measure defined on a background

S (=Jacobian) = physical quantity to ensure diffeomorphism invariance

which is Wess-Zumino action for conformal anomaly

physical against the name

Even at *t* = 0, *S* exists, that is Riegert action (=kinetic term of ϕ)

$$S_{\mathrm{R}} = -\frac{b_c}{(4\pi)^2}\int d^4x\sqrt{-\bar{g}}\left[2\phi\bar{\Delta}_4\phi + \left(\bar{G}_4 - \frac{2}{3}\bar{\nabla}^2\bar{R}\right)\phi\right]$$

The action has right sign :

4th-order conf. inv. op.

$$b_c = (N_S + 11N_F + 62N_A)/360 + 769/180$$

~ 10 for typical particle models

c.f. Liouville action in 2DQG

Asymptotic Background Freedom (=UV Dynamics)

This theory has conformal invariance as a "gauge symmetry" in the UV limit, that arises as subgroup of diffeomorphism invariance :

The theory = BRST CFT + perturbation by *t* (= deviation from CFT)

Riegert + Linearized Weyl actions (both have no $\hbar$)

BRST conformal symmetry

$$\hat{g}_{\mu\nu} \cong e^{2\omega(x)} \hat{g}_{\mu\nu}$$ while preserving the full metric $g_{\mu\nu}$ (= diff. inv.)

gauge equivalent

Conformal change of background does not change physics
= *background-metric independence* = no classical spacetime

BRST conformal symmetry makes all ghost modes unphysical

Infrared Dynamics

Classical spacetime will be realized when conformal dynamics disappear at low energy or long distance

Running coupling constant

(= deviation from conf. inv.)

WZ action of conformal anomaly, that is necessary to preserve diff. inv.

$$\Gamma_{\mathrm{W}} = \left[\frac{1}{t^2} - 2\underline{\beta_0 \phi} + \beta_0 \log\left(\frac{k^2}{\mu^2}\right) \right] \sqrt{-\bar{g}} \bar{C}^2_{\mu\nu\lambda\sigma}$$

$$= \frac{1}{\bar{t}^2(p)} \sqrt{-g} C^2_{\mu\nu\lambda\sigma}$$

(Here, ϕ is taken as a constant, for simplicity)

k = momentum measured on background

like comoving momentum

where

$$\bar{t}^2(p) = \frac{1}{\beta_0 \log(p^2/\Lambda^2_{\mathrm{QG}})}$$

$$\Lambda_{\mathrm{QG}} = \mu e^{-\frac{1}{2\beta_0 t^2}}$$

Physical momentum

New physical scale ($\mu \frac{d\Lambda_{\mathrm{QG}}}{d\mu} = 0$)

$$p^2 = \underline{\frac{k^2}{a^2}}$$ with $a = e^{\phi}$

diff. inv. in the full metric space

This indicates that WZ interactions can be incorporated by $t^2 \to \bar{t}^2(p)$

Value of Dynamical Scale

The value of $\Lambda_{\rm QG}$ has been determined from a scenario of inflation driven by quantum gravity dynamics only

Scale factor : $a(\tau) \propto e^{H_{\rm D}\tau}$

$$H_{\rm D} = M_{\rm P}\sqrt{8\pi^2/b_c}$$

($M_{\rm P} < H_{\rm D} < m_{\rm pl}$ for $b_c = 10$)

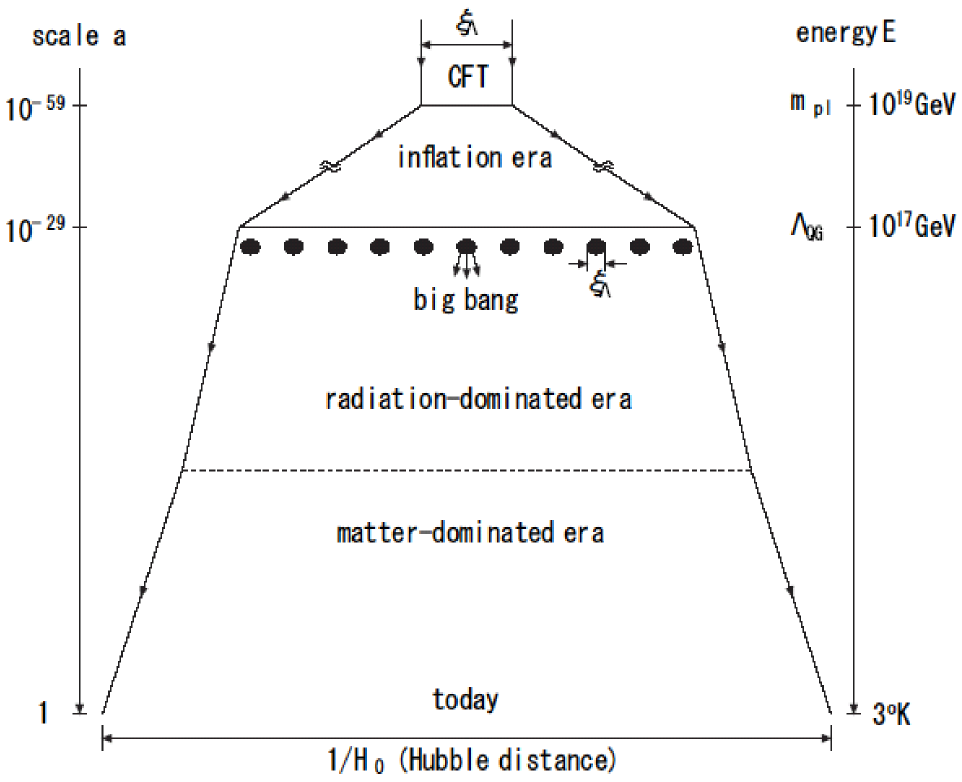
Number of e-foldings :

$$\mathcal{N}_e \simeq H_{\rm D}/\Lambda_{\rm QG} \sim 60$$

Amplitude of fluctuations :

$$\delta R/R \simeq \Lambda_{\rm QG}^2/12H_{\rm D}^2 \sim 10^{-5}$$

$$\boxed{\Lambda_{\rm QG} \simeq 10^{17}{\rm GeV}}$$



$1/H_0 \simeq 10^{59}\xi_\Lambda$ (~4000Mpc)

Correlation Length of QG

Correlation length of quantum gravity is given by

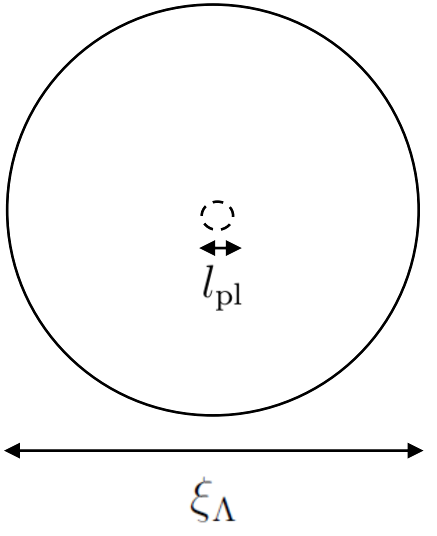
$$\xi_\Lambda = 1/\Lambda_{\rm QG} \simeq 2 \times 10^{-31}{\rm cm}$$

longer than the Planck length by two orders of magnitude

This will give a size of quantum gravity excitations

Inside, quantum gravity will be activated

Outside is ruled by Einstein gravity,
thus it has a Schwarzchild tail



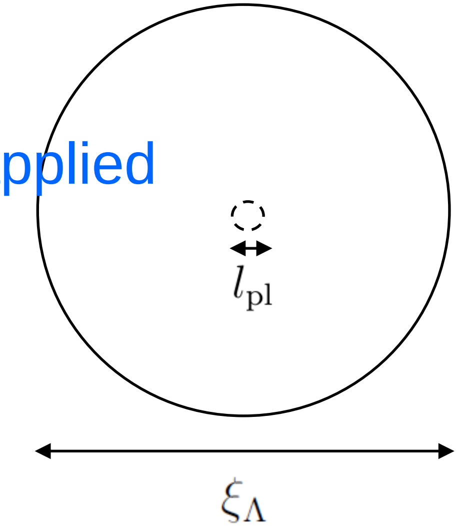
Dynamical Equations of Motion for Spherical Excitations

How To Describe Excitations

Introduce gravitational potentials

$$ds^2 = -(1+2\Psi)d\eta^2 + (1+2\Phi)d\mathbf{x}^2$$

If $\Psi,\ \Phi \ll 1$ inside, linear approximation can be applied



Outside is described by Schwarzschild solution:

$$\Phi = -\Psi = \frac{r_g}{2r} \qquad r_g = 2Gm$$: Schwarzschild radius

while the potentials do not monotonically increase inside

Necessary condition for linear approximation to be valid

Magnitude at edge must be small :

Radius = $R_h = \dfrac{\xi_\Lambda}{2}$, thus $2\Phi(R_h) = \dfrac{r_g}{R_h} \ll 1$

$$\Rightarrow \quad m \ll \frac{m_{\rm pl}^2}{4\Lambda_{\rm QG}} \simeq 25 m_{\rm pl} \qquad \text{from} \quad m_{\rm pl} \simeq 100\Lambda_{\rm QG}$$

Strategy To Find Solutions

The condition $m_{\rm pl} \gg \Lambda_{\rm QG}$ suggests that
if mass is not much larger than the Planck mass, there is an excitation satisfying the following situation:
"edge is in strong coupling, while the magnitude is still small everywhere"

Here, look for a solution satisfying such a situation

Actually, as shown below, I find

If the mass is several times the Planck mass,
it can be obtained as a solution of linearized equations of motion
for gravitational potentials incorporating strong coupling dynamics
under the spirit of mean field approximation

How To Include Strong Coupling Dynamics

The coefficient of the Riegert action receive higher-order corrections as

$$b_c \to b_c(1 - a_1 t^2 + \cdots) = b_c B(t)$$

Here, simply assume a summed-up form like

$$B(t) = \frac{1}{1 + a_1 t^2}$$

from a physical requirement that conformal dynamics disappear at $t \to \infty$

Further, incorporating running coupling effect

$$t^2 \to \bar{t}^2(p) = \frac{1}{\beta_0 \log(p^2 \xi_\Lambda^2)}$$

p is physical momentum : $p^2 = k^2 / e^{2\phi}$

As shown in Weyl sector, nonlinear terms from WZ interactions are contained in this form preserving diffeomorphism invariance

Approximation For Strong Coupling Dynamics

Let replacing *p* with a *r*-depedent mean field:

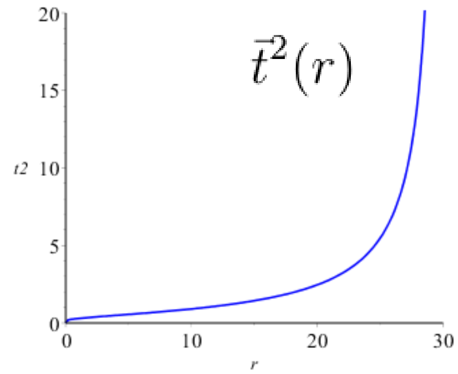
$$p \to \langle p \rangle = \frac{1}{2r}$$

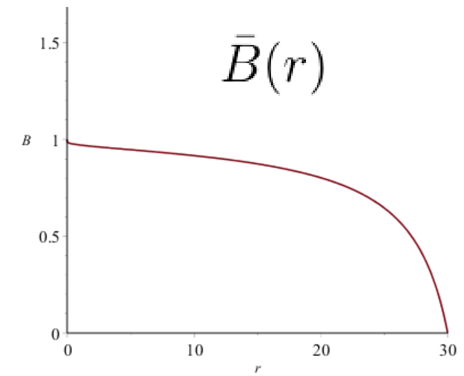
then

$$\bar{t}^2(r) = [\beta_0 \log(R_h^2/r^2)]^{-1}$$

and

$$\bar{B}(\bar{t}) = [1 + a_1 \bar{t}^2(r)]^{-1}$$





$$R_h = \xi_\Lambda/2 = 30 \ (H_\mathrm{D} = 1 \text{ unit})$$

Here, employ parameters adopted in the scenario of QG inflation

$$H_\mathrm{D}\,\xi_\Lambda = 60$$

from e-foldings and amplitude of power spectrum

$\leftrightarrow$ $\Lambda_{\mathrm{QG}} \simeq 10^{17}\mathrm{GeV}$

$$\beta_0 = 0.5 \quad a_1 = 0.1$$

Unlike b_c, these parameters are rather vague because they are built in the model of strong coupling dynamics. So, these are determined phenomenologically

In this way, nonlinear and nonperturbative effects are incorporated as a function of radial coordinate while maintaining linearity of potentials

Linearized Equations of motion

$$T_{\mu\nu} = T^{(4)}_{\mu\nu} + T^{\mathrm{EH}}_{\mu\nu} + T^{\mathrm{M}}_{\mu\nu} = 0$$

matter part = traceless perfect fluid

Coupled equations for potentials (without matter part) :

$$\not\partial^2 = \partial_r^2 + (2/r)\partial_r$$

Riegert

$$\frac{b_c}{8\pi^2}\bar{B}(\bar{t})\Big(-2\partial_\eta^4\Phi+\frac{10}{3}\partial_\eta^2\not\partial^2\Phi-\frac{4}{3}\not\partial^4\Phi+\frac{2}{3}\partial_\eta^2\not\partial^2\Psi -\frac{2}{3}\not\partial^4\Psi\Big)+M_\mathrm{P}^2(6\partial_\eta^2\Phi-4\not\partial^2\Phi-2\not\partial^2\Psi)=0$$

Einstein

trace eq.

Weyl

Riegert

$$\frac{b_c}{8\pi^2}\bar{B}(\bar{t})\Big(\frac{4}{3}\partial_\eta^2\not\partial^2\Phi-\frac{8}{9}\not\partial^4\Phi-\frac{4}{9}\not\partial^4\Psi\Big)+\frac{2}{\bar{t}^2}\Big(4\partial_\eta^2\not\partial^2\Phi-\frac{4}{3}\not\partial^4\Phi-4\partial_\eta^2\not\partial^2\Psi+\frac{4}{3}\not\partial^4\Psi\Big)-2M_\mathrm{P}^2(\not\partial^2\Phi+\not\partial^2\Psi)=0$$

Einstein

specific combination of spatial components

At the edge of excitation, running coupling constant diverges, so that $1/\bar{t}^2 \to 0$ and $\bar{B}(\bar{t}) \to 0$ at $r = R_h$

⇨ QG dynamics disappears at edge, leading to Einstein gravity

Energy Conservation Equation

Hamiltonian constraint (H = 0)

matter energy density

$$T_{00} = T_{00}^{(4)} + 2M_{\mathrm{P}}^2 \not\partial^2 \Phi + \rho = 0$$

where

$$T_{00}^{(4)} = \frac{b_c}{8\pi^2} \bar{B}(\bar{t}) \left(-\frac{2}{3} \partial_\eta^2 \not\partial^2 \Phi + \frac{4}{9} \not\partial^4 \Phi + \frac{2}{9} \not\partial^4 \Psi \right) + \frac{2}{\bar{t}^2} \left(-\frac{4}{3} \not\partial^4 \Phi + \frac{4}{3} \not\partial^4 \Psi \right)$$

Note that all gravitational terms contain spatial Laplacian

Mass of excitation

$$m = \int_{|\mathbf{x}| \le R_h} d^3\mathbf{x}\, T_{00}^{(4)}(\mathbf{x})$$

Static Excitation of Quantum Gravity

Static Equations of Motion

Introduce new variables

$$X = 2\Phi + \Psi \qquad\qquad Y = \Phi - \Psi$$

then

$$\begin{cases} \bar{B}(\bar{t})\,\partial\!\!\!/^4 X + 3H_{\mathrm{D}}^2\,\partial\!\!\!/^2 X = 0 \\ \dfrac{2}{\bar{t}^2}\,\partial\!\!\!/^4 Y - \dfrac{1}{2}M_{\mathrm{P}}^2\,\partial\!\!\!/^2 Y = 0 \end{cases}$$

$$H_{\mathrm{D}} = M_{\mathrm{P}}\sqrt{\frac{8\pi^2}{b_c}}$$

($M_{\mathrm{P}} < H_{\mathrm{D}} < m_{\mathrm{pl}}$ for $b_c = 10$)

completely decoupled, so can solve easily

From energy conservation equation, $\rho = 0$ is obtained

So, this excitation is a purely gravitational object

Boundary Conditions

Writing variables as

$$X(r) = \frac{r_g}{2r} f(r) \qquad Y(r) = \frac{r_g}{r} g(r) \qquad \text{then} \qquad \not{\partial}^2 X = \frac{r_g}{2r}\partial_r^2 f \qquad \not{\partial}^2 Y = \frac{r_g}{r}\partial_r^2 g$$

Then

$$\left\{ \begin{array}{rcl} \partial_r^4 f(r) + 3H_{\mathrm{D}}^2[1 + a_1 \vec{t}^2(r)]\partial_r^2 f(r) & = & 0 \\ \partial_r^4 g(r) - \dfrac{1}{4} M_{\mathrm{P}}^2 \vec{t}^2(r) \partial_r^2 g(r) & = & 0 \end{array} \right. \qquad \vec{t}^2(r) = [\beta_0 \log(R_h^2/r^2)]^{-1}$$

- Potentials and energy density do not diverge at origin

$$f(0) = 0 \quad g(0) = 0 \quad \partial_r^2 f(0) = 0 \quad \partial_r^2 g(0) = 0$$

- At edge, potentials are smoothly connected with Schwarzschild solution

$$f(R_h) = 1 \quad g(R_h) = 1 \quad \partial_r f(R_h) = 0 \quad \partial_r g(R_h) = 0$$

from $\Phi(R_h) = -\Psi(R_h)$
as Einstein eq. folds at edge

for smoothness

Behavior Around Origin

Near origin, $\vec{t}^2 \simeq 0$ and $\bar{B} \simeq 1$

writing $\zeta = \partial_r^2 f \quad \theta = \partial_r^2 g$ with b.c. $\zeta(0) = \theta(0) = 0$

Then

$$\begin{cases} \partial_r^2 \zeta + K^2 \zeta = 0 & K = \sqrt{3(1+a_1 t^2)} H_{\mathrm{D}} \simeq \sqrt{3} H_{\mathrm{D}} \\ \partial_r^2 \theta - L^2 \theta = 0 & L = M_{\mathrm{P}} t/2 \, (\ll 1) \end{cases}$$

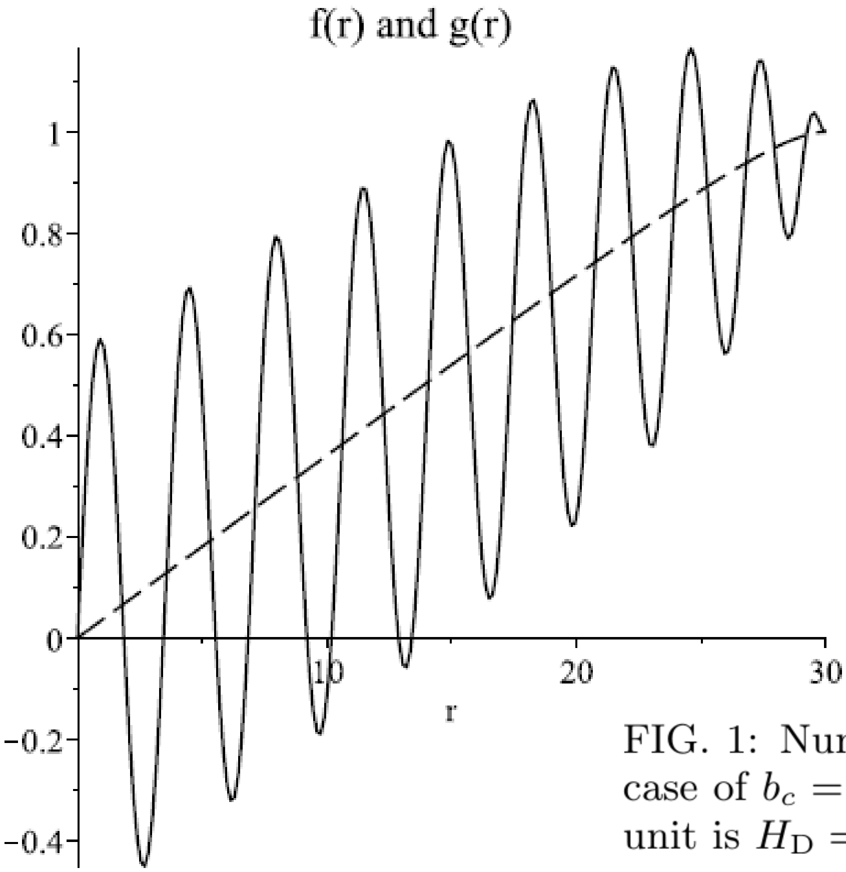
Solutions near origin :

$$\zeta \sim \sin(Kr) \qquad \theta \sim \sinh(Lr)$$

$$\Rightarrow \begin{cases} f(r) \simeq c \sin(\sqrt{3} H_{\mathrm{D}} r) + dr \\ g(r) \simeq d' r \end{cases}$$ (due to $L \ll 1$)

The coefficients cannot be determined unless the equations are completely solved by imposing the boundary conditions at the edge ➔ solve numerically

Numerical Solution



Around origin $(r \lesssim 5)$

$c = 0.543 \quad d = 0.037 \quad d' = 0.036$

$$f(r) \simeq c \sin(\sqrt{3} H_{\rm D} r) + dr$$

$$g(r) \simeq d' r$$

FIG. 1: Numerical results of f (solid) and g (dashed) in the case of $b_c = 10$, $\beta_0 = 0.5$, $a_1 = 0.1$, and $H_{\rm D}/\Lambda_{\rm QG} = 60$. The unit is $H_{\rm D} = 1$, then $m_{\rm pl} = 1.784$ and $M_{\rm P} = 0.356$.

The calculation is practically performed by setting the boundary condition at $r = R_h - \epsilon$ right inside the edge, and ϵ is brought close to zero until the result no longer changes. Here, $\epsilon = 0.0001$.

Gravitational Potentials

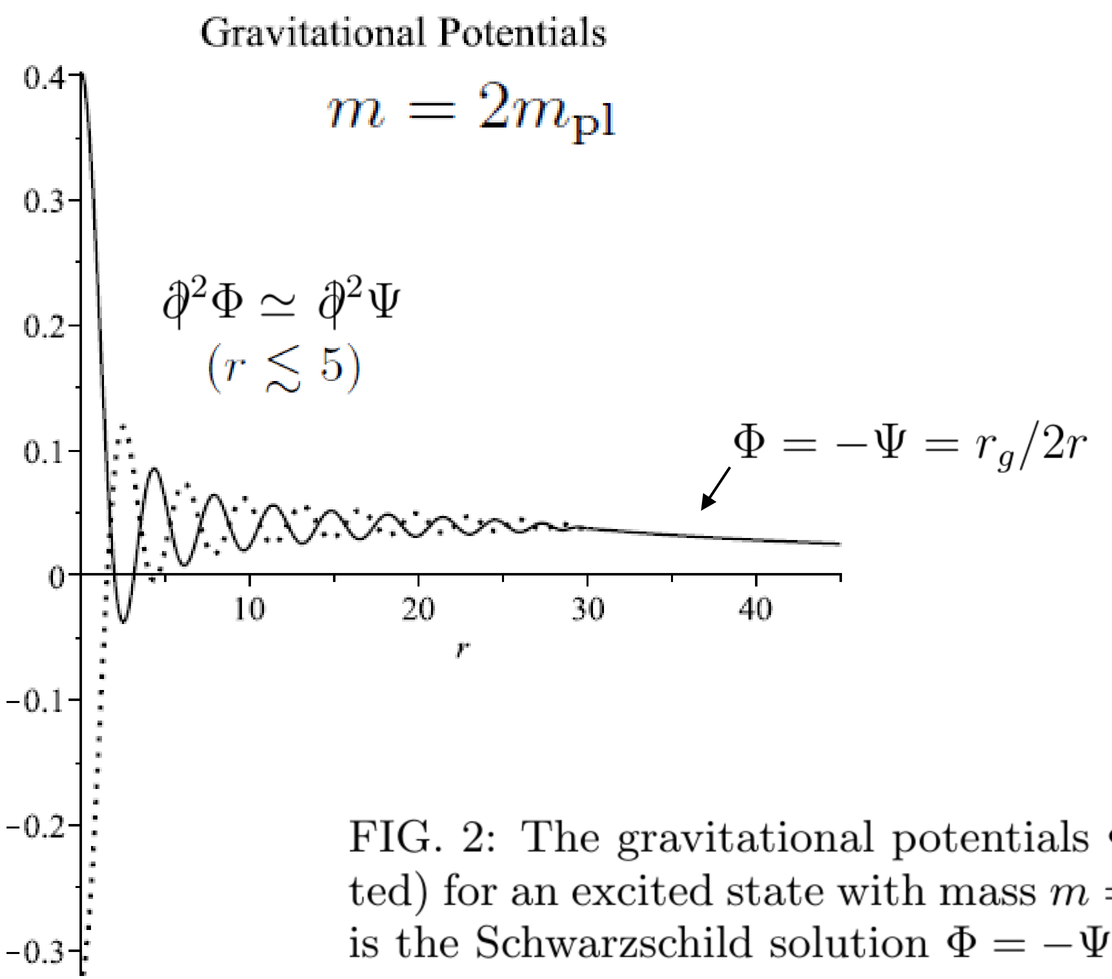


FIG. 2: The gravitational potentials Φ (solid) and $-\Psi$ (dotted) for an excited state with mass $m = 2m_{\rm pl}$. The tail $r \geq 30$ is the Schwarzschild solution $\Phi = -\Psi = r_g/2r$.

On Time Evolution

Equations Around Origin

Writing variables as

$$X(\eta, r) = \frac{r_g}{2r} F(\eta, r) \qquad Y(\eta, r) = \frac{r_g}{r} G(\eta, r)$$

Near origin, $\bar{t}^2 \simeq 0$ and $\bar{B} \simeq 1$

Coupled partial differential equations reduce to

$$\begin{cases} (\partial_\eta^2 - \partial_r^2)^2 F(\eta, r) + 2\partial_\eta^2(\partial_\eta^2 - \partial_r^2) G(\eta, r) \\ -3H_{\mathrm{D}}^2 \left[(\partial_\eta^2 - \partial_r^2) F(\eta, r) + 2\partial_\eta^2 G(\eta, r) \right] = 0 \\ \qquad\qquad (3\partial_\eta^2 - \partial_r^2)\, \partial_r^2 G(\eta, r) = 0 \end{cases}$$

Initial conditions

$$F(0, r) = f(r) \qquad G(0, r) = g(r)$$ where *f* and *g* are static solutions

Boundary conditions at origin

$$F(\eta, 0) = 0 \quad G(\eta, 0) = 0 \quad \partial_r^2 F(\eta, 0) = 0 \quad \partial_r^2 G(\eta, 0) = 0$$

Stability of Static Excitation Near Origin

General solution near origin

$$\begin{cases} F(\eta, r) = \{c + b\sin(\sqrt{3}H_{\mathrm{D}}\eta) + b'[\cos(\sqrt{3}H_{\mathrm{D}}\eta) - 1]\}\sin(\sqrt{3}H_{\mathrm{D}}r) \\ \qquad\qquad + \tilde{e}(\eta)r - 2\tilde{d}(\eta)r \\ G(\eta, r) = \tilde{d}(\eta)\, r \end{cases}$$

$c,\ d,\ d'$ are given as initial conditions

where $\tilde{d}(0) = d' \quad \tilde{e}(0) = d + 2d'$

$b,\ b'$ are unknown constants

F and *G* are not monotonic functions of time, except unknown time functions $\tilde{d}(\eta),\ \tilde{e}(\eta)$

because all gravitational terms in *H* involve spatial Laplacian

However, these two do not affect changes in energy equation
Therefore, gravitational energy does not shift to matter density ρ monotonically

⇩

Excitation seems to be kept stable without energy changing into matters

Of course, there is a possibility that energy leaks from edge ➔ future issue

Conclusion and Discussion

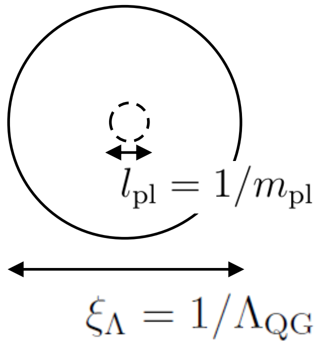
Conclusion

The dynamics of the asymptotically background-free quantum gravity begin to work at the energy scale $\Lambda_{\rm QG} \simeq 10^{17}{\rm GeV}$ below the Planck scale.

The excited state examined here has several times Planck mass, sufficiently larger than $\Lambda_{\rm QG}$, so that quantum gravity will be activated inside.

A static and spherical excitation with Schwarzschild tail was constructed as a solution of linearized equations-of-motion for gravitational potentials incorporating running coupling effects.

It can be regarded as a particle, not BH, when viewed from the outside because its diameter ξ_Λ is larger than the horizon size $2r_g$.



Discussion

If the mass is smaller than Λ_{QG}, quantum gravity will not be activated and no state will be excited. The coupling constant will remain large everywhere and the assumption that it is running will not be valid.

On the other hand, a macroscopic object with a semi-classical horizon whose size is larger than ξ_Λ looks like a black hole. It will undergo black hole evaporation, and may eventually leave the small excited state as a remnant.

It is thought that many excited states were generated in the early universe. Primordial black holes could be formed by using them as seeds.
If the state is actually stable or long-lived, it can be a candidate for dark matter as a purely gravitational object.

Physical Minimal Length

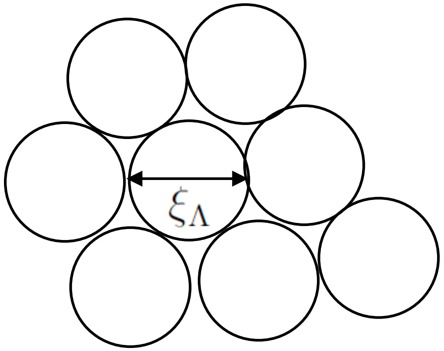
Correlation length can be regarded as minimal length

We cannot measure distance shorter than correlation length precisely because quantum gravity is activated and distance is greatly fluctuating as it was shown here

Distance is “practically” discretized by correlation length

$$\xi_\Lambda = 1/\Lambda_{\rm QG} \simeq 0.2 \times 10^{-30}{\rm cm}$$

without breaking diffeomorphism invariance, unlike lattice-type models



In initial universe before inflation, spacetime would have been filled densely with excitations like bubbles

Origin of Primordial Fluctuations

Quantum gravity Inflation indicates that the universe expand about $10^{59}\ (= 10^{30} \times 10^{29})$ from the beginning to today, that is

$$10^{59} \times \xi_\Lambda \sim \frac{1}{H_0}$$ Hubble distance (~4000Mpc)

The universe we see today was originated from an excitation of quantum gravity with the size of correlation length ξ_Λ

Inside of the excitation is almost scale invariant

➔ Origin of the scale invariance of primordial power spectrum
Edge has been observed as sharp-falling of low multipoles in CMB

Initial power spectrum is

$$P_\varphi(k) = \frac{1}{2b_c}$$ where $$\left\langle (\varphi(\mathbf{x}) - \varphi(\mathbf{y}))^2 \right\rangle = 2\int \frac{dk}{k} P_\varphi(k) \left(1 - \frac{\sin k|\mathbf{x}-\mathbf{y}|}{k|\mathbf{x}-\mathbf{y}|} \right)$$

fluctuation of conformal factor field

Revisit Meaning of Diffeomorphism Invariance

Hamiltonian and momentum constraints
= Wheeler-Dewitt equations
= Equations of motion for gravitational field (=Schwinger-Dyson eqs)

$$\langle T_{\mu\nu} \rangle = 0$$ involving quantum corrections

This means that Hamiltonian is normal-ordered quantity
and zero-point energy disappears !

This is a general property that correct theory of quantum gravity must satisfy

Hence
- Origin of primordial fluctuations is not zero-point energy
  - ➔ quantum gravity excitations
- There is no original cause of cosmological constant problem
  - ➔ cosmological constant is a physical scale, namely RG invariant

Evolution of the universe is described as process in which
energy state changes while preserving $H = 0$

The fact that E-H action is unbounded from below
is crucial to meet non-trivial dynamics

Appendix

Hamiltonian and Momentum Constraints

The whole action of quantum gravity

$$\mathcal{I} = S + I$$

Schwinger-Dyson equations

$$\int [d\phi\, dh\, df]_{\hat{g}} \frac{\delta}{\delta\phi} e^{i\mathcal{I}} = i\sqrt{-\hat{g}} \Big\langle \mathbf{T}^{\lambda}{}_{\lambda} \Big\rangle \quad = \quad 0$$

$$\int [d\phi\, dh\, df]_{\hat{g}} \frac{\delta}{\delta h^{\nu}{}_{\mu}} e^{i\mathcal{I}} = \frac{i}{2}\sqrt{-\hat{g}} \Big\langle \Big(\mathbf{T}^{\mu}{}_{\nu} - \delta^{\mu}{}_{\nu} \mathbf{T}^{\lambda}{}_{\lambda}/4 \Big) \Big\rangle \quad = \quad 0$$

$$\Rightarrow \qquad \langle \mathbf{T}_{\mu\nu} \rangle = 0$$

where

$$\delta\mathcal{I} = \frac{1}{2}\int d^4x \sqrt{-g}\, T^{\mu\nu}\delta g_{\mu\nu} = \int d^4x \sqrt{-\hat{g}} \left[\mathbf{T}^{\lambda}{}_{\lambda}\delta\phi + \frac{1}{2}\mathbf{T}^{\mu}{}_{\nu}\delta h^{\nu}{}_{\mu} \right]$$

This indicates that EM tensor is normal-ordered quantity and zero-point energy vanishes

Diffeomorphism Invariance

$$\delta_\xi g_{\mu\nu} = g_{\mu\lambda}\nabla_\nu \xi^\lambda + g_{\nu\lambda}\nabla_\mu \xi^\lambda$$

ξ^μ : gauge d.o.f.

Metric field is expanded as

$$g_{\mu\nu} = e^{2\phi}\bar{g}_{\mu\nu} \qquad \bar{g}_{\mu\nu} = (\hat{g}e^{th})_{\mu\nu} = \hat{g}_{\mu\lambda}\left(\delta^\lambda{}_\nu + th^\lambda{}_\nu + \frac{t^2}{2}(h^2)^\lambda{}_\nu + \cdots\right)$$

conformal factor
<u>exactly</u>

background

traceless tensor field
<u>perturbatively</u>

gauge-fixed later

Diff. transf. is then decomposed as

$$\delta_\xi \phi = \xi^\lambda \partial_\lambda \phi + \frac{1}{4}\hat{\nabla}_\lambda \xi^\lambda$$

$$\delta_\xi h_{\mu\nu} = \frac{1}{t}\left(\hat{\nabla}_\mu \xi_\nu + \hat{\nabla}_\nu \xi_\mu - \frac{1}{2}\hat{g}_{\mu\nu}\hat{\nabla}_\lambda \xi^\lambda\right) + \xi^\lambda \hat{\nabla}_\lambda h_{\mu\nu} + \frac{1}{2}h_{\mu\lambda}\left(\hat{\nabla}_\nu \xi^\lambda - \hat{\nabla}^\lambda \xi_\nu\right)$$
$$+\frac{1}{2}h_{\nu\lambda}\left(\hat{\nabla}_\mu \xi^\lambda - \hat{\nabla}^\lambda \xi_\mu\right) + o(t)$$

two modes completely decoupled!

Physical States and Positivity

Physical states are classified by imposing BRST conf. inv.

$$Q_{BRST}|\mathrm{phys}\rangle = 0 \qquad \begin{cases} \delta_{\mathrm{B}}\phi = c^{\mu}\partial_{\mu}\phi + \frac{1}{4}\partial_{\mu}c^{\mu} \\ \delta_{\mathrm{B}}h_{\mu\nu} = c^{\lambda}\partial_{\lambda}h_{\mu\nu} + \frac{1}{2}h_{\mu\lambda}\left(\partial_{\nu}c^{\lambda} - \partial^{\lambda}c_{\nu}\right) + \frac{1}{2}h_{\nu\lambda}\left(\partial_{\mu}c^{\lambda} - \partial^{\lambda}c_{\mu}\right) \end{cases}$$

The existence of this symmetry *in the UV limit* is a crucial difference from other HD QG models

All ghost modes are not gauge invariant *!*

Physical states are given by *real primary scalars* with conf. dim. 4 satisfying unitarity bound, when viewed as CFT on $\hat{g}_{\mu\nu}$

Furthermore, the action is bounded from below (unlike E-H action)
Therefore, the path integral is well-defined

➔ the reality of physical states is guaranteed

Background-metric Independence

This QG model has background-metric indep. in UV limit ($t = 0$)

Outline of the proof

First, notice that the theory is invariant under a simultaneous shift: $\phi \to \phi - \omega$ and $\hat{g}_{\mu\nu} \to e^{2\omega}\hat{g}_{\mu\nu}$,
because it preserves the full metric field $g_{\mu\nu} = e^{2\phi}\hat{g}_{\mu\nu}$

Further, conformal-factor field is an integral variable in QG,
and now it is treated exactly without introducing its own coupling

Thus, the measure is invariant under the shift $\phi' = \phi - \omega$ as

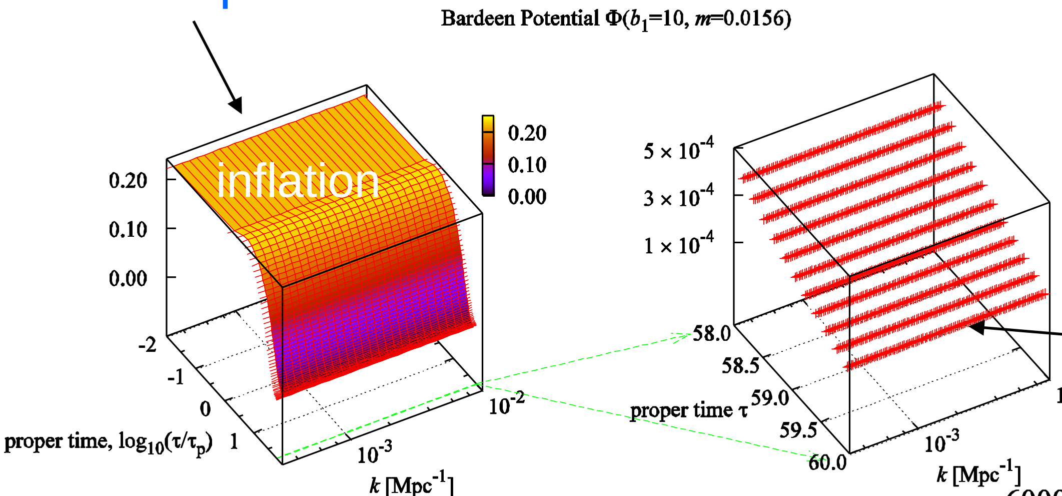
$$\int_{-\infty}^{\infty}[d\phi'] = \int_{-\infty}^{\infty}[d\phi]$$

Consequently, the theory becomes invariant under the conformal change such as

$$Z(e^{2\omega}\hat{g}) = Z(\hat{g})$$

Evolution of Fluctuations from CFT to CMB

Scale-inv. spectrum at Planck time
with amp. = 1/b ~ 10^{-2}



From Planck length to
cosmological distance

$$10^{59} = 10^{30} + 10^{29}$$

inflation Friedmann

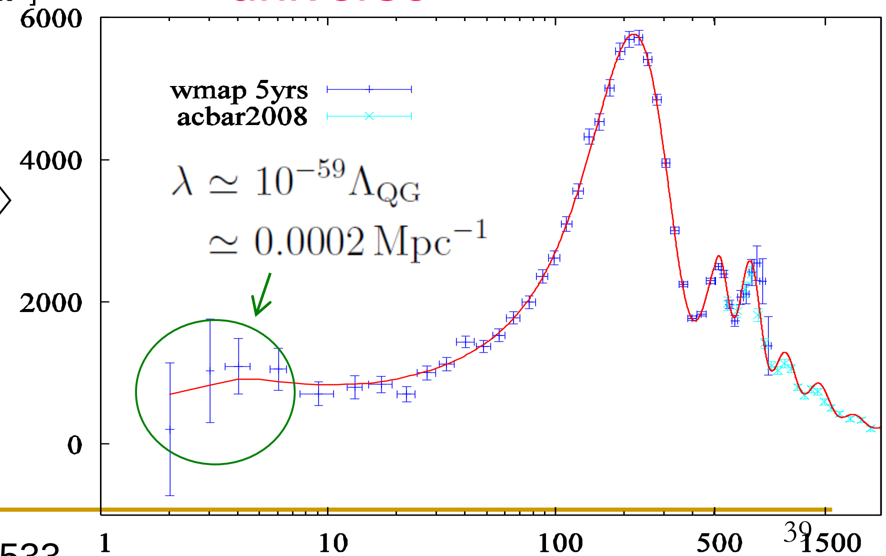
spectrum at transition point
= initial condition of Friedmann
universe

CMB spectrum is computed
using CMBFAST Fortran code

Initial condition is then set to
be almost scale invariant:

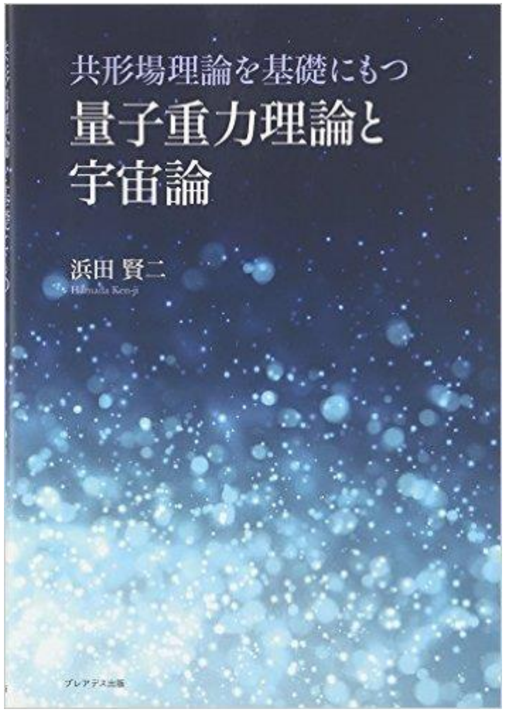
$$P_s(k) = A_s\left(\frac{k}{m}\right)^{n_s-1+v/\log(k^2/\lambda^2)}\quad,\quad n_s = 1$$

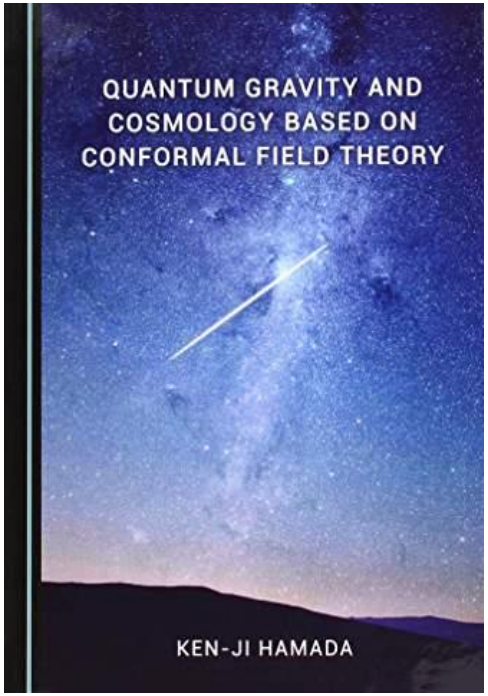
~10^{-9} scale invariant



K.H., S.Horata and T. Yukawa, PRD 81(2010)083533

Books





「共形場理論を基礎にもつ
量子重力理論と宇宙論」
（プレアデス出版、2016）

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